Tables of Facts from Logic

by Robert Heckendorn
Computer Science Dept., University of Idaho

Tautologies (Logical Identities)

Commutative Property:	$p \wedge q$	$\leftrightarrow$	$q \wedge p$
	$p \vee q$	$\leftrightarrow$	$q \vee p$
	$p \oplus q$	$\leftrightarrow$	$q \oplus p$
Associative Property:	$(p \wedge q) \wedge r$	$\leftrightarrow$	$p \wedge (q \wedge r)$
	$(p \vee q) \vee r$	$\leftrightarrow$	$p \vee (q \vee r)$
	$(p \oplus q) \oplus r$	$\leftrightarrow$	$p \oplus (q \oplus r)$
Distributive Property:	$p \wedge (q \vee r)$	$\leftrightarrow$	$(p \wedge q) \vee (p \wedge r)$
	$p \wedge (q \oplus r)$	$\leftrightarrow$	$(p \wedge q) \oplus (p \wedge r)$
	$p \vee (q \wedge r)$	$\leftrightarrow$	$(p \vee q) \wedge (p \vee r)$
	$p \vee (q \rightarrow r)$	$\leftrightarrow$	$(p \vee q) \rightarrow (p \vee r)$
	$p \rightarrow (q \wedge r)$	$\leftrightarrow$	$(p \rightarrow q) \wedge (p \rightarrow r)$
	$p \rightarrow (q \vee r)$	$\leftrightarrow$	$(p \rightarrow q) \vee (p \rightarrow r)$
De Morgan's Laws:	$\sim(p \wedge q)$	$\leftrightarrow$	$\sim p \vee \sim q$
	$\sim(p \vee q)$	$\leftrightarrow$	$\sim p \wedge \sim q$
	$\sim(p \oplus q)$	$\leftrightarrow$	$\sim p \oplus q$
	$\sim(p \oplus q)$	$\leftrightarrow$	$p \oplus \sim q$
	$\sim(p \rightarrow q)$	$\leftrightarrow$	$p \wedge \sim q$
Transposition (Contrapositive):	$p \rightarrow q$	$\leftrightarrow$	$\sim q \rightarrow \sim p$
	$p \oplus q$	$\leftrightarrow$	$\sim p \oplus \sim q$
Involution (Double Negation):	p	$\leftrightarrow$	$\sim\sim p$
Material Implication:	$p \rightarrow q$	$\leftrightarrow$	$\sim p \vee q$
Material Equivalence:	$p \leftrightarrow q$	$\leftrightarrow$	$(p \rightarrow q) \wedge (q \rightarrow p)$
Partial Associativity:	$a \rightarrow (b \rightarrow c)$	$\leftrightarrow$	$b \rightarrow (a \rightarrow c)$
Exportation:	$(p \wedge q) \rightarrow r$	$\leftrightarrow$	$p \rightarrow (q \rightarrow r)$
Absurdity:	$(p \rightarrow q) \wedge (p \rightarrow \sim q)$	$\leftrightarrow$	$\sim p$
	$(\sim p \vee q) \wedge (p \vee r)$	$\leftrightarrow$	$(p \wedge q) \vee (\sim p \wedge r)$
Destructive Distribution:	$a \wedge (\sim a \vee b)$	$\leftrightarrow$	$a \wedge b$
	$a \wedge (\sim a \oplus b)$	$\leftrightarrow$	$a \wedge b$
	$a \wedge (a \rightarrow b)$	$\leftrightarrow$	$a \wedge b$
	$a \vee (\sim a \wedge b)$	$\leftrightarrow$	$a \vee b$
	$a \vee (\sim a \rightarrow b)$	$\leftrightarrow$	$a \vee b$
	$a \vee (a \oplus b)$	$\leftrightarrow$	$a \vee b$
	$a \rightarrow (\sim a \vee b)$	$\leftrightarrow$	$a \rightarrow b$
	$a \rightarrow (\sim a \oplus b)$	$\leftrightarrow$	$a \rightarrow b$
	$(a \vee b) \rightarrow b$	$\leftrightarrow$	$a \rightarrow b$
	$(a \oplus b) \rightarrow b$	$\leftrightarrow$	$a \rightarrow b$
	$(a \rightarrow b) \rightarrow a$	$\leftrightarrow$	b
	$(p \rightarrow r) \vee (q \rightarrow r)$	$\leftrightarrow$	$((p \wedge q) \rightarrow r)$
	$(p \rightarrow r) \wedge (q \rightarrow r)$	$\leftrightarrow$	$((p \vee q) \rightarrow r)$

Implications

Modus Ponens:	$(p \to q)\,;\ \ p$	$\to$	q
Modus Tollens:	$(p \to q)\,;\ \ {\sim}q$	$\to$	${\sim}p$
Hypothetical Syllogism:	$(p \to q)\,;\ \ (q \to r)$	$\to$	$(p \to r)$
Disjunctive Syllogism:	$(p \vee q)\,;\ \ {\sim}p$	$\to$	q
Constructive Delemma:	$(p \to q) \wedge (r \to s)\,;\ \ p \vee r$	$\to$	$(q \vee s)$
Destructive Delemma:	$(p \to q) \wedge (r \to s)\,;\ \ {\sim}q \vee {\sim}s$	$\to$	$({\sim}p \vee {\sim}r)$
Simplification:	$p \wedge q$	$\to$	p
Conjunction:	$p\,;\ \ q$	$\to$	$p \wedge q$
Addition:	p	$\to$	$(p \vee q)$
Law of Resolution:	$(p \vee q) \wedge ({\sim}p \vee r)$	$\to$	$q \vee r$
	${\sim}(p \to q)$	$\to$	$q \to p$
	$p \to q$	$\to$	$p \wedge q$
	$p \to q$	$\to$	$(p \wedge r) \to q$
	$p \to q$	$\to$	$(p \wedge r) \to (q \wedge r)$
	$p \to q$	$\to$	$(p \vee r) \to (q \vee r)$

Equivalents in Packing and Unpacking Bit Fields

In contrast to previous sections this section deals with operatators that work on bitstrings. Specifically, the symbols $\vdash$ and $\dashv$ are pack and unpack bit fields. $(p \dashv m)$ means to unpack the string p using mask m. For example: $(1011 \dashv 110011)$ gives 100011. The length of p must be the same as the number of 1 bits in m. $(p \vdash m)$ means to pack the string p using mask m. For example: $(1011011 \dashv 1100111)$ gives 10011. The length of p must be the same as the length of m. The resulting string has the same number of bits as there are 1 bits in m. All logic operators are bitwise operators.

Associative Property:	$((p \dashv m) \dashv n)$	$=$	$(p \dashv (m \dashv n))$
SemiAssociative Property:	$((p \vdash m) \vdash n)$	$=$	$(p \vdash (m \dashv n))$
Negation of Pack:	$\overline{p \vdash m}$	$=$	$\overline{p} \vdash m$
Negation of Unpack:	$\overline{p \dashv m}$	$=$	$\overline{m} \vee (\overline{p} \dashv m)$
	$\overline{p \dashv m}$	$=$	$\overline{m} \oplus (\overline{p} \dashv m)$
Inverse Property:	$(p \dashv m) \vdash m$	$=$	p
SemiInverse Property:	$(p \vdash m) \dashv m$	$=$	$p \wedge m$
Distributive Property:	$(p \vee q) \dashv m$	$=$	$(p \dashv m) \vee (q \dashv m)$
	$(p \oplus q) \dashv m$	$=$	$(p \dashv m) \oplus (q \dashv m)$
	$(p \wedge q) \dashv m$	$=$	$(p \dashv m) \wedge (q \dashv m)$
	$(m \wedge p) \vdash p$	$=$	$m \vdash p$
	$(m \dashv p) \wedge p$	$=$	$m \dashv p$
	$(p \dashv (m \vee n)) \vdash m$	$=$	$p \vdash (m \vdash (m \vee n))$